Hong Kong Mathematics Olympiad (1990 – 91)
Heat Event (Group)
香港数学竞赛 (1990 – 91)
初赛项目 (团体)

1. Find the unit digit of 1357^{7890}.
求 1357^{7890} 的个位数。

2. If $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\cdots+\frac{1}{2450}=\frac{x}{100}$, find x.
若 $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\cdots+\frac{1}{2450}=\frac{x}{100}$，求 x。

3. $\frac{a}{3}$, $\frac{b}{4}$ and $\frac{c}{6}$ are three proper fractions in their simplest form, where a, b and c are positive integers. If c is added to the numerator of each fraction, then the sum of the fractions formed will be equal to 6. Find the value of $a+b+c$.
$\frac{a}{3}$ 、 $\frac{b}{4}$ 及 $\frac{c}{6}$ 是三个化至最简的真分数，其中 a 、 b 及 c 是正整数。如果这三个分数的分子都加上 c，则所得三个分数的和是 6。求 $a+b+c$ 的值。

4. Study the Pascal's triangle shown below:

Row 1						1						
Row 2					1		1					
Row 3				1		2		1				
Row 4			1		3		3		1			
Row 5		1		4		6		4		1		
Row 6	1		5		10		10		5		1	
						⋮						

Find the sum of all the numbers from Row 1 to Row 15.
细读下列之帕斯卡三角形

第 1 行						1						
第 2 行					1		1					
第 3 行				1		2		1				
第 4 行			1		3		3		1			
第 5 行		1		4		6		4		1		
第 6 行	1		5		10		10		5		1	
						⋮						

求由第 1 行至第 15 行所有数的总和。

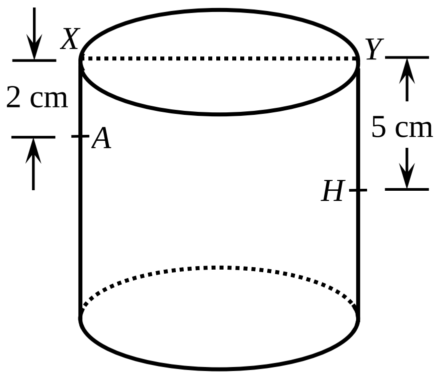
5. In the multiplication □□□ × □□ = □□ × □□ = 5568, each of the above boxes represents an integer from 1 to 9. If the integers for the nine boxes above are all different, find the number represented by □□□.
在下列乘法算式中
□□□ × □□ = □□ × □□ = 5568，
每一方格代表由 1 至 9 的一个整数。若以上九个方格所代表的九个整数都不相同，求 □□□ 所代表的整数。

6. Find the remainder when $1997^{1990} - 1991$ is divided by 1996.
求 $1997^{1990} - 1991$ 被 1996 除所得的余数。

7. Find the least positive integral value of n such that $\sqrt{n} - \sqrt{n-1} < \frac{1}{80}$.
求满足不等式 $\sqrt{n} - \sqrt{n-1} < \frac{1}{80}$ 的 n 的最小正整数值。

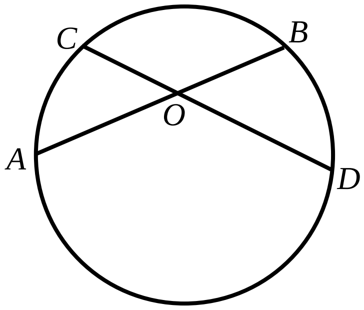
8. One of the solutions of the equation $32x + 59y = 3259$ in positive integers is given by $(x, y) = (100, 1)$. It is known that there is exactly one more pair of positive integers a, b ($a \neq 100$ and $b \neq 1$) such that $32a + 59b = 3259$. Find a.
方程 $32x + 59y = 3259$ 的其中一组正整数解为 $(x, y) = (100, 1)$。现知仅有另一组正整数 a、b $(a \neq 100, b \neq 1)$ 使得 $32a + 59b = 3259$，求 a。

9. In Figure 1, XY is a diameter of a cylindrical glass, 48 cm in base circumference. On the outside is an ant at A, 2 cm below X and on the inside is a small drop of honey at H, 5 cm below Y. If the length of the shortest path for the ant to reach the drop of honey is x cm, find x. (Neglect the thickness of the glass.)
在图 1 中，XY 是圆柱形玻璃杯的直径，杯底的圆周是 48 cm。杯外 A 点处 (在 X 之下 2 cm) 有一蚁，杯内 H 点处 (在 Y 之下 5 cm) 有一小滴蜜糖。若蚁行至蜜糖的最短路线长 x cm，求 x。(杯的厚度可略去不计。)



(Figure 1)(图 1)

10. In Figure 2, two chords AOB, COD cut at O. If the tangents at A and C meet at X, the tangents at B and D meet at Y and $\angle AXC = 130°$, $\angle AOD = 120°$, $\angle BYD = k°$, find k.
在图 2 中，弦 AOB、 COD 相交于 O。若过 A 的切线与过 C 的切线相交于 X，过 B 的切线与过 D 的切线相交于 Y，且 $\angle AXC = 130°$、$\angle AOD = 120°$、$\angle BYD = k°$，求 k。



(Figure 2)(图 2)